

# Time-dependent interacting dark energy and transient acceleration

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## Abstract

We investigate cosmological scenarios in which dark matter and dark energy interact with a time-dependent phenomenological form. Starting from simple and extending to more complicated ansatzes, we obtain analytical expressions for the evolutions of the deceleration and the various density parameters. We find that depending on the choices of the model parameters, in the far future the universe can either result to a dark-energy domination, in which the late-time acceleration is permanent, or to a dark-matter domination after passing through a transient accelerating phase.

PACS numbers: 98.80.-k, 95.36.+x

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## I. INTRODUCTION

Recent cosmological observations support that the universe is experiencing an accelerated expansion at late times [1]. In principle there are at least two ways to explain such a behavior, apart from the simple consideration of a cosmological constant. The first approach is to modify the gravitational sector itself (see [2] for a review and references therein), obtaining a modified cosmological dynamics. The other approach is to modify the content of the universe introducing the dark energy sector, which can be based on a canonical scalar field (quintessence) [3, 4], a phantom field [5], or the combination of quintessence and phantom fields in a unified scenario [6] (see [7] for a review).

However, the dynamical nature of dark energy introduces a new cosmological problem, namely why the energy densities of dark energy and dark matter are nearly equal today although they scale independently during the expansion history. The elaboration of this “coincidence” problem led to the consideration of generalized versions of the above models with the inclusion of a coupling between dark energy and dark matter. Thus, various forms of “interacting” dark energy models [8–12] have been constructed in order to fulfill the observational requirements.

Due to the lack of information about dark energy and dark matter, thus in the aforementioned phenomenological models the interaction terms in general were assumed to be proportional to the energy densities and their derivatives, and to the Hubble parameter. However, such forms restrict the time-dependence of the interaction term to a small and peculiar subclass of possibilities, so a different approach was followed in [13, 14]. Instead of giving particular forms of the interaction term, the effect of the interaction on the evolution of dark matter was explicitly shown by the parameter  $\epsilon(a)$  through the solution  $\rho_{dm} = \rho_{dm0} a^{-3+\epsilon(a)}$ , with  $a$  the scale factor. Thus one can obtain interesting cosmological behavior by choosing the form of  $\epsilon(a)$ , .

In the present work, we combine the above two approaches, that is we generalize the interaction terms of [8–12], allowing for a general time-dependence of the form [13, 14]. It proves that even very simple forms can alleviate the coincidence problem, and lend the cosmic acceleration a transient character. It was shown that it is problematic to define a set of observable quantities analogous to the S-matrix for string theory in an eternally accelerating universe due to the existence of event horizon [15]. Therefore the existence of

transient acceleration not only explains the current cosmic acceleration, but also avoids the above mentioned problem for string theory.

The plan of the work is as follows: In section II we construct the time-dependent interacting dark energy scenario, starting from a simple form (subsection II A), and extending the analysis in a more general interaction form (subsection II B). Finally, section III is devoted to the summary of the results.

## II. TIME-DEPENDENT INTERACTING DARK ENERGY

Let us now construct the time-dependent interacting dark energy scenario. Throughout the work we consider a flat Friedmann-Robertson-Walker metric of the form  $ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2$ . The evolution equations for the dark energy and dark matter (considered as dust for simplicity) densities read as

$$\dot{\rho}_{dm} + 3H\rho_{dm} = Q, \quad (1)$$

$$\dot{\rho}_{de} + 3H(\rho_{de} + p_{de}) = -Q, \quad (2)$$

with  $p_{de}$  the dark energy pressure,  $Q$  the interaction term,  $H \equiv \dot{a}/a$  the Hubble parameter, and dot denoting differentiation with respect to  $t$ . Therefore,  $Q > 0$  corresponds to energy transfer from dark energy to dark matter, while  $Q < 0$  corresponds to energy transfer from dark matter to dark energy. In general, we may consider a general interaction between scalar field and matter [16]

$$\mathcal{L} = \sqrt{-g} \left[ -\frac{\mathcal{R}}{2\kappa^2} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) \right] - \sqrt{-\tilde{g}}\mathcal{L}_m(\psi, \tilde{g}_{\mu\nu}), \quad (3)$$

where  $\kappa^2 = 8\pi G$  and

$$\tilde{g}_{\mu\nu} = C(\phi)g_{\mu\nu} + D(\phi)\partial_\mu\phi\partial_\nu\phi.$$

Then the interaction  $Q$  takes the form

$$Q = -\frac{C' - 2D(3H\dot{\phi} + V' + C'\dot{\phi}^2/C) + D'\dot{\phi}^2}{2(C + D(\rho_m - \dot{\phi}^2))}\rho_m\dot{\phi}.$$

The above interaction is a generalization of scalar tensor theory of gravity written in Einstein frame with  $C = e^{2\beta\kappa\phi}$  and  $D = 0$ . Dependent on the choice of  $C$  and  $D$ , we may derive

desired interaction form  $Q$ . The system of dynamical equation closes by considering one of the Friedmann equations:

$$H^2 = \frac{\kappa^2}{3} (\rho_{de} + \rho_{dm} + \rho_b), \quad (4)$$

$$\dot{H} = -\frac{\kappa^2}{2} (\rho_{de} + p_{de} + \rho_{dm} + \rho_b), \quad (5)$$

where we have also included the dust baryon density (one can also straightforwardly include radiation too).

### A. Simplest model

Lets us now determine the form of the interaction term  $Q$ . As we mentioned in the introduction, we start by the forms of [8–12], allowing for a general time-dependence of the form [13, 14]. In particular, in the literature the interacting forms are chosen as  $Q = \alpha_0 \dot{\rho}_{de}$  and  $Q = 3\beta_0 H \rho_{de}$  with constant  $\alpha_0$  and  $\beta_0$  [8–12], we generalize it to be

$$Q = 3\beta(a) H \rho_{de}, \quad (6)$$

with a simple power-law ansatz for  $\beta(a)$ , namely:

$$\beta(a) = \beta_0 a^\xi. \quad (7)$$

Substituting this interaction form into Eq. (2) we obtain

$$\rho_{de} = \rho_{de0} a^{-3(1+w_0)} \cdot \exp \left[ \frac{3\beta_0(1-a^\xi)}{\xi} \right], \quad (8)$$

where the integration constant  $\rho_{de0}$  is value of dark energy at present, and for simplicity we have considered the dark energy equation-of-state parameter  $w \equiv p_{de}/\rho_{de}$  to be a constant  $w_0$ . Substituting Eq. (8) into Eq. (1), we get the dark matter energy density,

$$\rho_{dm} = f(a) \rho_{dm0}, \quad (9)$$

where

$$f(a) \equiv \frac{1}{a^3} \left\{ 1 - \frac{\Omega_{de0}}{\Omega_{dm0}} \frac{3\beta_0 a^{-3w_0} e^{\frac{3\beta_0}{\xi}}}{\xi} \cdot \left[ a^\xi E_{\frac{3w_0}{\xi}} \left( \frac{3\beta_0 a^\xi}{\xi} \right) - a^{3w_0} E_{\frac{3w_0}{\xi}} \left( \frac{3\beta_0}{\xi} \right) \right] \right\}, \quad (10)$$

with  $\rho_{dm0}$  the present-day dark-matter density, and  $E_n(z) = \int_1^\infty t^{-n} e^{-xt} dt$  the usual exponential integral function. Note however that Eq. (9) is an analytic expression, while in [13, 14] the corresponding expressions were left as integrals and were elaborated numerically. Obviously, in the case of non-interaction, that is for  $\beta_0 = 0$ , Eq. (9) recovers the standard result  $\rho_{dm} = \rho_{dm0}/a^3$ . For the special case  $\xi = 0$ , the energy densities of the dark sectors are

$$\rho_{de} = \rho_{de0} a^{-3(1+w_0+\beta_0)}, \quad (11)$$

$$\rho_{dm} = \rho_{dm0} a^{-3} \left[ 1 + \frac{\Omega_{de0}}{\Omega_{dm0}} \frac{\beta_0}{w_0 + \beta_0} (1 - a^{-3(w_0+\beta_0)}) \right]. \quad (12)$$

It is now easy to use the Friedmann equation (4) to define the dimensionless Hubble parameter, namely

$$E^2(z) \equiv \frac{H^2}{H_0^2} = \Omega_{b0} a^{-3} + \Omega_{dm0} f(a) + \Omega_{de0} a^{-3(1+w_0)} e^{\frac{3\beta_0(1-a^\xi)}{\xi}}, \quad (13)$$

where  $\Omega_i \equiv \kappa^2 \rho_i / 3H_0^2$ , and  $\Omega_{i0} \equiv \kappa^2 \rho_{i0} / 3H_0^2$  are the present values of energy density parameters. Therefore, from Eqs. (8), (9) and (13) we can straightforwardly obtain the evolution of the density parameters as

$$\Omega_b(a) = \frac{a^{-3}}{a^{-3} + A f(a) + B a^{-3(1+w_0)} e^{\frac{3\beta_0(1-a^\xi)}{\xi}}} \quad (14)$$

$$\Omega_{dm}(a) = \frac{f(a)}{A^{-1} a^{-3} + f(a) + A^{-1} B a^{-3(1+w_0)} e^{\frac{3\beta_0(1-a^\xi)}{\xi}}} \quad (15)$$

$$\Omega_{de}(a) = \frac{a^{-3(1+w_0)} e^{\frac{3\beta_0(1-a^\xi)}{\xi}}}{B^{-1} a^{-3} + A B^{-1} f(a) + a^{-3(1+w_0)} e^{\frac{3\beta_0(1-a^\xi)}{\xi}}}, \quad (16)$$

where  $A = \Omega_{dm0}/\Omega_{b0}$  and  $B = \Omega_{de0}/\Omega_{b0}$ . Finally, we can easily calculate analytically the deceleration parameter

$$q \equiv -\frac{\ddot{a}}{aH^2} = -1 + \frac{3}{2} \left[ \frac{\Omega_b + \Omega_m + (1+w_0)\Omega_{de}}{\Omega_b + \Omega_m + \Omega_{de}} \right], \quad (17)$$

leading to

$$q = -1 + \frac{3}{2} \left[ \frac{a^{-3} + A f(a) + B(1+w_0) a^{-3(1+w_0)} e^{\frac{3\beta_0(1-a^\xi)}{\xi}}}{a^{-3} + A f(a) + B a^{-3(1+w_0)} e^{\frac{3\beta_0(1-a^\xi)}{\xi}}} \right]. \quad (18)$$

For the special case  $\xi = 0$ , using Eqs. (11) and (12), we get

$$q = \frac{1}{2} + \frac{w_0 \Omega_{de0}}{w_0 \Omega_{de0}/(w_0 + \beta_0) + (1 - w_0 \Omega_{de0}/(w_0 + \beta_0))a^{3(w_0 + \beta_0)}}. \quad (19)$$

So for  $\xi = 0$ , when  $\beta_0 > -w_0 - 1/2$ , the cosmic acceleration is transient.

Up to now we derived analytical expressions for the evolution of the various density parameters, and the deceleration parameter, with only the present density parameter values and the dark energy equation-of-state parameter as free parameters. It is therefore straightforward to construct their evolution graphs, using the observational values  $\Omega_{de0} \approx 0.72$ ,  $\Omega_{dm0} \approx 0.24$ ,  $\Omega_{b0} \approx 0.04$  [17], and setting the present scale factor value to 1.

In the upper left panel of Fig. 1 we plot the evolution of the various density parameters with  $\beta_0 = -0.02$ ,  $w_0 = -0.9$  and  $\xi = -0.8$ , corresponding to energy transfer from dark matter to dark energy. Due to the energy transfer from dark matter to dark energy, despite the fact that the energy transfer decreases as time passes by ( $\xi$  is negative), we obtain the expected result of complete dark energy domination in the future. This result is independent of the values of  $\xi$  and  $w_0$ , and a positive  $\xi$  would just bring the dark energy domination earlier. In the lower left panel of Fig. 1 we depict the corresponding evolution of the deceleration parameter. Clearly, we can see that in this scenario, the late-time cosmic acceleration is permanent.

In the upper right panel of Fig. 1 we present the evolutions of the various density parameters with  $\beta_0 = 0.12$ ,  $w_0 = -1.1$  and  $\xi = 1.2$ . It is clear that the cosmic acceleration is transient. Because positive  $\beta_0$  corresponds to the energy transfer from dark energy to dark matter and positive  $\xi$  means increasing energy transfer as the universe evolves, so dark matter will finally become the dominant component.

In the phantom case  $w_0 < -1$ , we find that the interaction can not only save the universe from a Big Rip [18], but also lead to a dark matter domination. Additionally, in the lower left panel of Fig. 1 we plot the evolution of the deceleration parameter. From these plots we can clearly see that the present acceleration of the universe is transient when both  $\beta_0$  and  $\xi$  are positive. This is a very interesting result from the phenomenological point of view, and one of the main results of the present work. The result of transient acceleration is quite general for interacting models with more and more energy transfer from dark energy to dark matter.

In the lower right panel of Fig. 1, we show the evolutions of the effective equation of state

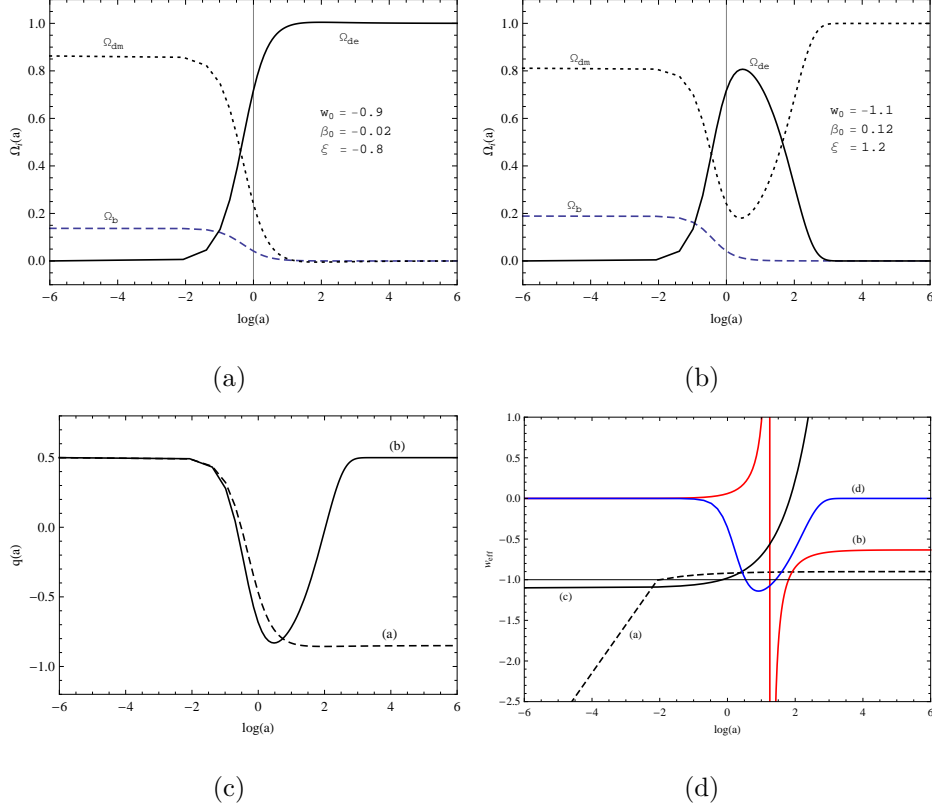


FIG. 1: The results for the simplest interacting model  $Q = 3\beta_0 a^\xi H \rho_{de}$ . Upper left panel (a): The evolutions of the various density parameters for  $\beta_0 = -0.02$ ,  $\xi = -0.8$  and  $w_0 = -0.9$ . Upper right panel (b): The evolutions of the various density parameters for  $\beta_0 = 0.12$ ,  $\xi = 1.2$  and  $w_0 = -1.1$ . Lower left panel (c): The corresponding evolutions of the deceleration parameter  $q$ . Line (a) is for the parameters  $\beta_0 = -0.02$ ,  $\xi = -0.8$  and  $w_0 = -0.9$  and line (b) is for the parameters  $\beta_0 = 0.12$ ,  $\xi = 1.2$  and  $w_0 = -1.1$ . Lower right panel (d): the evolutions of the effective equation of state for dark energy (lines (a) and (c)) and dark matter (lines (b) and (d)). Lines (a) and (b) are for the parameters  $\beta_0 = -0.02$ ,  $\xi = -0.8$  and  $w_0 = -0.9$  and lines (c) and (d) are for the parameters  $\beta_0 = 0.12$ ,  $\xi = 1.2$  and  $w_0 = -1.1$ .

parameters  $w_{eff}$  for both dark energy and dark matter. We see that the effective equation of state parameter of dark energy becomes positive in the future due to the energy transfer from dark energy to dark matter when transition acceleration happens, and dark matter behaves like dark energy in the future due to the energy transfer from dark matter to dark energy when eternal acceleration happens.

## B. General scenarios

In this subsection we extend the previous analysis in more general time-dependent interacting scenarios. In particular, we add  $\alpha(a)\dot{\rho}_{de}$  and consider

$$Q = 3\beta(a)H\rho_{de} + \alpha(a)\dot{\rho}_{de}, \quad (20)$$

with a simple power-law ansatz for  $\alpha(a)$ :

$$\alpha(a) = \alpha_0 a^\eta, \quad (21)$$

and the same power-law form (7) for  $\beta(a)$ .

Substituting this interaction form into Eq. (2), we obtain

$$\begin{aligned} \rho_{de} = & \rho_{de0} a^{-3(1+w_0)} \left( \frac{1 + \alpha_0 a^\eta}{1 + \alpha_0} \right)^{\frac{3(1+w_0)}{\eta}} \\ & \times \exp \left\{ 3\beta_0 \xi^{-1} \left[ {}_2F_1 \left( 1, \frac{\xi}{\eta}; \frac{\eta + \xi}{\eta}; -\alpha_0 \right) - a^\xi {}_2F_1 \left( 1, \frac{\xi}{\eta}; \frac{\eta + \xi}{\eta}; -a^\eta \alpha_0 \right) \right] \right\}, \end{aligned} \quad (22)$$

where  ${}_2F_1(a, b; c; z)$  is the usual hypergeometric function, and again we consider the dark energy equation-of-state parameter to be a constant  $w_0$ . Note that this expression coincides with Eq. (8) when  $\alpha_0 = 0$ , as expected.

Unfortunately, inserting the above solution (22) into Eqs. (20) and (1), does not lead to an analytical expression for  $\rho_{dm}$ . Since in the present paper we desire to provide analytical results, so we restrict ourselves in the simpler scenario

$$\alpha(a) = \beta(a) = \alpha_0 a^\xi. \quad (23)$$

In this case, Eq. (22) becomes

$$\rho_{de} = \rho_{de0} a^{-3(1+w_0)} \left( \frac{1 + \alpha_0 a^\xi}{1 + \alpha_0} \right)^{\frac{3w_0}{\xi}}, \quad (24)$$

and the solution for dark matter is

$$\rho_{dm} = g(a)\rho_{dm0}, \quad (25)$$

where

$$\begin{aligned} g(a) \equiv & \frac{1}{a^3} \left\{ 1 - \frac{\Omega_{de0}}{\Omega_{dm0}} \left( \frac{3\alpha_0 w_0}{3w_0 - \xi} \right) \times \right. \\ & \left. \left[ {}_2F_1 \left( 1, 1; 2 - \frac{3w_0}{\xi}; -\alpha_0 \right) - a^{\xi-3w_0} \left( \frac{1 + \alpha_0 a^\xi}{1 + \alpha_0} \right)^{\frac{3w_0}{\xi}} {}_2F_1 \left( 1, 1; 2 - \frac{3w_0}{\xi}; -\alpha_0 a^\xi \right) \right] \right\}. \end{aligned}$$



Note that in the case of no-interaction, that is for  $\alpha_0 = 0$ , Eq. (25) gives the standard result  $\rho_{dm} = \rho_{dm0}/a^3$ . In fact, substituting Eq. (23) into Eq. (20), the interacting form becomes  $Q = -3\alpha(a)w_0H\rho_{de}/(1 + \alpha(a))$ . So the form of interaction is the same as Eq. (6) except that now  $\beta(a) = -\alpha(a)w_0/(1 + \alpha(a))$ . The general scenario is just a general form of the simplest scenario with  $\beta(a) = -\alpha(a)w_0/(1 + \alpha(a))$ . For  $\xi > 0$ ,  $\beta(a) \rightarrow -w_0$  when  $a \rightarrow \infty$ . It becomes the special case  $\xi = 0$  of the simplest scenario. Since  $\beta(a) = -w_0 > -w_0 - 1/2$  when  $a \rightarrow \infty$ , so the acceleration is a transient one for  $\xi > 0$ . For the special case  $\xi = 0$ , we get the energy densities of dark sectors,

$$\rho_{de} = \rho_{de0}a^{-3(1+w_0/(1+\alpha_0))}, \quad (26)$$

$$\rho_{dm} = \rho_{dm0}a^{-3} \left[ 1 + \alpha_0 \frac{\Omega_{de0}}{\Omega_{dm0}} (a^{-3w_0/(1+\alpha_0)} - 1) \right]. \quad (27)$$

The dimensionless Hubble parameter reads:

$$E^2(z) \equiv \frac{H^2}{H_0^2} = \Omega_{b0}a^{-3} + \Omega_{dm0}g(a) + \Omega_{de0}a^{-3(1+w_0)} \left( \frac{1 + \alpha_0 a^\xi}{1 + \alpha_0} \right)^{\frac{3w_0}{\xi}}. \quad (28)$$

Thus, Eqs. (24), (25) and (28) give

$$\Omega_b(a) = \frac{a^{-3}}{a^{-3} + Ag(a) + Ba^{-3(1+w_0)} \left( \frac{1+\alpha_0 a^\xi}{1+\alpha_0} \right)^{\frac{3w_0}{\xi}}} \quad (29)$$

$$\Omega_{dm}(a) = \frac{g(a)}{A^{-1}a^{-3} + g(a) + A^{-1}Ba^{-3(1+w_0)} \left( \frac{1+\alpha_0 a^\xi}{1+\alpha_0} \right)^{\frac{3w_0}{\xi}}} \quad (30)$$

$$\Omega_{de}(a) = \frac{a^{-3(1+w_0)} \left( \frac{1+\alpha_0 a^\xi}{1+\alpha_0} \right)^{\frac{3w_0}{\xi}}}{B^{-1}a^{-3} + AB^{-1}g(a) + a^{-3(1+w_0)} \left( \frac{1+\alpha_0 a^\xi}{1+\alpha_0} \right)^{\frac{3w_0}{\xi}}}, \quad (31)$$

where again  $A = \Omega_{dm0}/\Omega_{b0}$  and  $B = \Omega_{de0}/\Omega_{b0}$ . Finally, using Eq. (17) the deceleration parameter is written as

$$q = -1 + \frac{3}{2} \left[ \frac{a^{-3} + Ag(a) + B(1+w_0)a^{-3(1+w_0)} \left( \frac{1+\alpha_0 a^\xi}{1+\alpha_0} \right)^{\frac{3w_0}{\xi}}}{a^{-3} + Ag(a) + Ba^{-3(1+w_0)} \left( \frac{1+\alpha_0 a^\xi}{1+\alpha_0} \right)^{\frac{3w_0}{\xi}}} \right]. \quad (32)$$

For the special case  $\xi = 0$ , using Eqs. (26) and (27), we get

$$q = \frac{1}{2} + \frac{w_0 \Omega_{de0}}{(1 + \alpha_0) \Omega_{de0} + (1 - (1 + \alpha_0) \Omega_{de0}) a^{3w_0/(1+\alpha_0)}}. \quad (33)$$

So for the special case  $\xi = 0$ , when  $\alpha_0 > -2w_0 - 1$  or  $\alpha_0 < -1$ , the cosmic acceleration is transient.

Since we obtain analytical expressions for the evolutions of the various density parameters and of the deceleration parameter, with the present density parameter values and the dark energy equation-of-state parameter as free parameters, we proceed to present the corresponding evolutions.

In the upper left panel of Fig. 2 we plot the evolution of the density parameters with  $\alpha_0 = -0.01$ ,  $w_0 = -0.9$  and  $\xi = -0.5$ , corresponding to energy transfer from dark matter to dark energy.

As expected, the energy transfer from dark matter to dark energy despite the fact that it is decreasing ( $\xi$  is negative), leads to complete dark energy domination in the future. This result is independent of the value of  $\xi$  and  $w_0$ , and a positive  $\xi$  or more negative  $w_0$  would just bring the dark energy domination earlier. In the lower left panel of Fig. 2 we depict the corresponding evolution of the deceleration parameter. As we can see, the late-time cosmic acceleration is permanent.

In the upper right panel of Fig. 2, we plot evolution of the density parameters with  $\alpha_0 = 0.1$ ,  $w_0 = -1.1$  and  $\xi = 1.6$ . As expected, the energy transfer from dark energy to dark matter leads to dark matter domination in the future. Furthermore, in the lower left panel of Fig. 2 we present the evolution of the deceleration parameter. From these plots we can clearly see that the current acceleration of the universe is transient for  $\xi > 0$ . Note that the transient acceleration happens for  $\xi > 0$  even though  $\alpha_0$  is negative, and  $\alpha_0 > -2w_0 - 1$  or  $\alpha_0 < -1$  when  $\xi = 0$ .

In the lower right panel of Fig. 2, we show the evolutions of the effective equation of state parameters  $w_{eff}$  for both dark energy and dark matter. We see that in terms of the effective equation of state parameter, dark energy behaves like dark matter in the future due to the energy transfer from dark energy to dark matter when transition acceleration happens, and dark matter behaves like dark energy in the future due to the energy transfer from dark matter to dark energy when eternal acceleration happens.

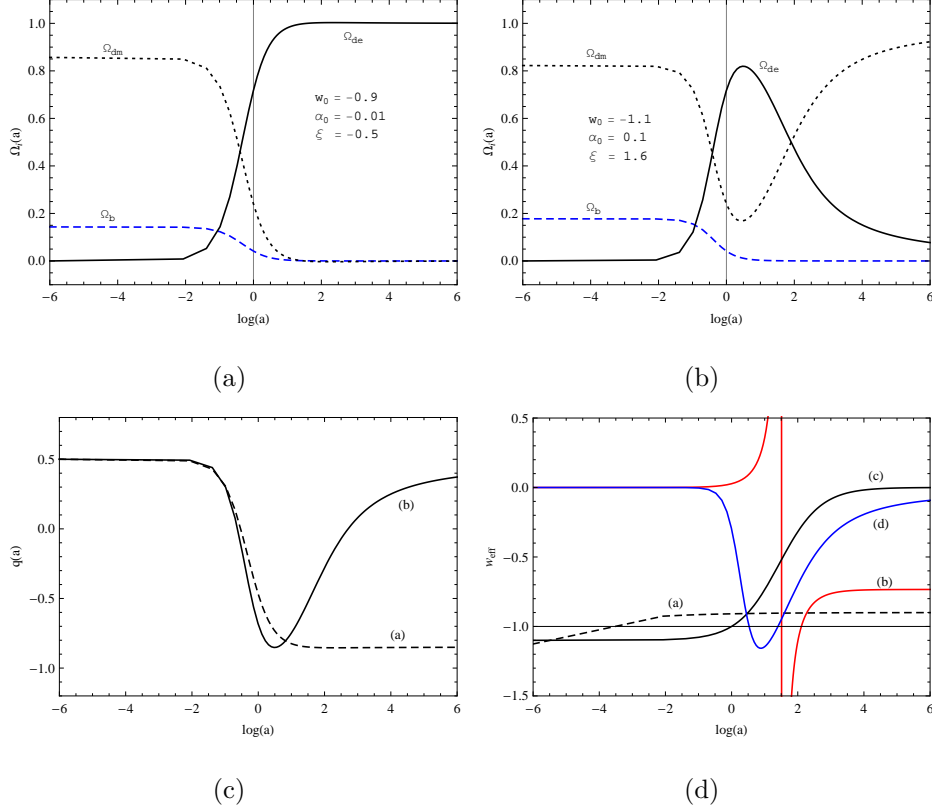


FIG. 2: The results for the simplest interacting model  $Q = \alpha_0 a^\xi (\dot{\rho}_{de} + 3H\rho_{de})$ . Upper left panel (a): The evolutions of the various density parameters for  $\alpha_0 = -0.01$ ,  $\xi = -0.5$  and  $w_0 = -0.9$ . Upper right panel (b): The evolutions of the various density parameters for  $\alpha_0 = 0.1$ ,  $\xi = 1.6$  and  $w_0 = -1.1$ . Lower left panel (c): The corresponding evolutions of the deceleration parameter  $q$ . Line (a) is for the parameters  $\alpha_0 = -0.01$ ,  $\xi = -0.5$  and  $w_0 = -0.9$  and line (b) is for the parameters  $\alpha_0 = 0.1$ ,  $\xi = 1.6$  and  $w_0 = -1.1$ . Lower right panel (d): the evolutions of the effective equation of state for dark energy (lines (a) and (c)) and dark matter (lines (b) and (d)). Lines (a) and (b) are for the parameters  $\alpha_0 = -0.01$ ,  $\xi = -0.5$  and  $w_0 = -0.9$  and lines (c) and (d) are for the parameters  $\alpha_0 = 0.1$ ,  $\xi = 1.6$  and  $w_0 = -1.1$ .

### III. CONCLUSIONS

In the present work we investigated cosmological scenarios in which dark matter and dark energy interact with each other by a time-dependent interaction form, and we obtained analytical expressions for the evolutions of the deceleration and the various density parameters. The resulting cosmological behavior proves to be very interesting.

In the case of a simple time-dependent interaction of the form  $Q = 3\beta_0 a^\xi H\rho_{de}$ , for

negative  $\beta_0$ , the energy transfer from dark matter to dark energy leads to a dark energy dominated universe, independent of the values of  $\xi$  and  $w_0$ . In this case the late-time cosmic acceleration is permanent. However, for positive  $\beta_0$  and  $\xi$ , the energy transfer from dark energy to dark matter leads to a late-time dark-matter domination. In this case the current cosmic acceleration presents a transient character, alleviating the coincidence problem. For the special case  $\xi = 0$ , transient acceleration happens when  $\beta_0 > -w_0 - 1/2$ .

In the case of a more general interaction form  $Q = 3\alpha_0 a^\xi H \rho_{de} + \alpha_0 a^\xi \dot{\rho}_{de}$ , we obtain similar results. That is, for negative  $\alpha_0$  the universe is led to a complete dark energy domination, independently of  $w_0$  and  $\xi$ , with a permanent late-time acceleration. On the other hand, for positive  $\xi$  the energy transfer from dark energy to dark matter may lead to a dark matter domination in the far future, with a transient cosmic acceleration. For the special case  $\xi = 0$ , the transient acceleration happens for  $\alpha_0 > -2w_0 - 1$  or  $\alpha_0 < -1$ .

The transient character of the cosmic acceleration is very interesting from both the phenomenological and observational point of view, and moreover it can offer an explanation for the recent indications that the current cosmic acceleration is slowing down [19]. The time-dependent interaction, starting from the specific phenomenological forms examined above, offers an alternative way of obtaining the transient acceleration comparing to other mechanisms proposed in the literature [14, 20].

## Acknowledgments

This work was partially supported by the National Basic Science Program (Project 973) of China under grant No. 2010CB833004, the NNSF project of China under grant Nos. 10935013 and 11175270, CQ CSTC under grant No. 2009BA4050 and CQ CMEC under grant Nos. KJTD201016 and KJ110523.

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